Lesson Plan: The Geometry of the Gnomon (Mathematics)

Subject

Mathematics (Geometry, Angles, Measurement, Basic Trigonometry - High School)

Grade Level

Middle School (Grades 7-8) & High School (Grades 9-12)

Time Allotment

2-3 class periods

Learning Objectives

Upon completion of this lesson, students will be able to:

- Identify and measure various types of angles related to shadow casting.
- Understand the concept of a gnomon and its function in a sundial.
- Apply geometric principles (similar triangles, angle relationships) to analyze shadow lengths.
- (High School) Utilize basic trigonometric ratios (tangent) to calculate unknown lengths or angles.
- Appreciate the mathematical precision required in memorial design.

Materials

- Images/diagrams of the Kentucky Vietnam Veterans Memorial gnomon.
- Protractor, ruler, measuring tape.
- Flashlights or a strong light source.
- Small upright objects (pencils, dowel rods, blocks) to act as mini-gnomons.
- Graph paper or plain paper.
- Calculators (especially for high school).
- (Optional) Access to online sundial simulators or geometry software.

Procedure

Day 1: Introduction to Gnomons & Shadow Play (45-60 minutes)

1. **Introduction to the Memorial (10 minutes):**
    - Show images of the Kentucky Vietnam Veterans Memorial, specifically highlighting the gnomon (the central pillar that casts the shadow).
    - Explain that the gnomon's shadow interacts with the names on the wall throughout the year. Ask students: "How do you think this shadow is designed

to hit specific names on specific days?"

2. **What is a Gnomon? (15 minutes):**
    - Define "gnomon" as the part of a sundial that casts the shadow.
    - Discuss how the angle of the sun changes throughout the day and year, and how this affects shadow length and direction.
    - **Activity:** "Shadow Play" - In small groups, give students a flashlight and a small upright object. Have them experiment with casting shadows on a piece of paper.
        - Ask them to move the flashlight to change the shadow's length and direction.
        - Have them draw the object and its shadow, marking the angle of the light source.
3. **Basic Angle Relationships (20-35 minutes):**
    - Draw a simple diagram on the board: a vertical stick, the ground, and a line representing a sun ray hitting the top of the stick and extending to the end of the shadow.
    - Identify the right angle formed by the stick and the ground.
    - Introduce/review angles of elevation/depression as they relate to the sun's rays and the shadow.
    - Discuss similar triangles: If you have two objects of different heights, how would their shadows relate at the same time of day?
    - **Assignment:** Students draw a diagram of their "Shadow Play" experiment, labeling the gnomon, shadow, and relevant angles.

Day 2: Calculating Shadows & Angles (45-60 minutes)

1. **Review & Connect to Memorial (10 minutes):**
    - Briefly review the concepts from Day 1.
    - Reiterate that the Kentucky memorial's gnomon is precisely angled to coincide with the latitude of Frankfort, allowing its shadow to track names on specific dates.
2. **Shadow Length Calculations (Middle School) / Trigonometry (High School) (30-40 minutes):**
    - **Middle School Focus:**
        - Using the concept of similar triangles, provide scenarios where students calculate unknown shadow lengths or gnomon heights given other measurements.
        - Example problem: "If a 3-foot stick casts a 4-foot shadow, how long would the shadow be for a 6-foot pole at the same time?"
    - **High School Focus:**

        - Introduce the tangent function (opposite/adjacent).
        - Explain how tan(angle of elevation) = (gnomon height) / (shadow length).
        - Provide problems where students calculate:
            - Shadow length given gnomon height and sun's angle.
            - Gnomon height given shadow length and sun's angle.
            - The sun's angle given gnomon height and shadow length.
        - Use the actual dimensions of the Kentucky memorial's gnomon (or simplified versions) for problems. (e.g., Gnomon height ~ 30 feet, calculate shadow length at a given angle).
3. **Hands-on Measurement (15-20 minutes, if applicable):**
    - If possible, take students outside with protractors and measuring tapes.
    - Have them measure the height of a flagpole or another tall object and its shadow length at a specific time.
    - Using their measurements, have them calculate the angle of elevation of the sun.
    - Compare results and discuss potential sources of error.

Day 3: Designing a Simple Sundial (Optional, 45-60 minutes)

1. **Sundial Basics (15 minutes):**
    - Explain that the memorial's gnomon acts as a sundial, marking not just dates but also time (though the primary focus is dates).
    - Discuss how a sundial works: the gnomon casts a shadow, and lines on the dial plate mark the hours.
    - Crucially, the gnomon must be aligned with the Earth's axis (pointing North Star in the Northern Hemisphere) and its angle with the ground must match the local latitude.
2. **Design a Simple Sundial (30-45 minutes):**
    - Challenge students to design a simple horizontal sundial for their location (using Frankfort's latitude for consistency: approx. 38.2° N).
    - They need to:
        - Determine the angle of their gnomon.
        - Draw the dial plate and estimate where hour lines would be based on the changing shadow position throughout the day. (Simplified: just mark cardinal directions and noon).
    - Emphasize the need for geometric precision in their design.
    - **Extension:** Students could construct a simple paper sundial.

Assessment

- **Worksheet/Problem Set:** Evaluate understanding of angle relationships, shadow

calculations, and (for high school) trigonometry application.
- **Sundial Design (Optional):** Graded on geometric accuracy and understanding of principles.
- **Participation:** Engagement in discussions and hands-on activities.

Extension Activities

- **Virtual Sundial Tour:** Explore online interactive sundial exhibits.
- **Historical Context:** Research the history of sundials and gnomons across different cultures.
- **Programming Challenge:** Write a simple program to calculate shadow lengths given height and sun angle.
- **Scale Model:** Create a scaled model of the Kentucky Vietnam Veterans Memorial gnomon and its shadow interaction.